Stable isochronal synchronization of mutually coupled chaotic lasers

Einat Klein,[1] Noam Gross,[1] Michael Rosenbluh, Wolfgang Kinzel,[2] Lev Khaykovich,[1] and Ido Kanter[1]
[1]*Department of Physics, Bar-Ilan University, Ramat-Gan, 52900 Israel*
[2]*Institut für Theoretische Physik, Universität Würzburg, Am Hubland 97074 Würzburg, Germany*

(Received 11 October 2005; revised manuscript received 28 November 2005; published 14 June 2006)

The dynamics of two mutually coupled chaotic diode lasers are investigated experimentally and numerically. By adding self-feedback to each laser, stable isochronal synchronization is established. This stability, which can be achieved for symmetric operation, is essential for constructing an optical public-channel cryptographic system. The experimental results on diode lasers are well described by rate equations of coupled single mode lasers.

DOI: 10.1103/PhysRevE.73.066214 PACS number(s): 05.45.Vx, 42.55.Px, 42.65.Sf

A semiconductor laser, when subjected to optical feedback, displays chaotic behavior [1–3]. This phenomenon has been investigated over the last two decades with synchronization between two chaotic lasers attracting much recent interest because of its applicability to secret communication [4–7]. Different configurations, such as delayed optoelectronic feedback [8–10] or coherent optical injection [11,12,14] have been suggested for the synchronization of two semiconductor lasers. The coupling between the lasers is accomplished in a unidirectional [5,8,9] or bidirectional [10,11,14] fashion, leading to different kinds of synchronization phenomena. Recently it was shown that it might be possible to use the synchronization of two symmetric chaotic lasers for cryptographic key-exchange protocols, whereby secret messages can be transmitted over public channels without using any previous secrets [15]. A necessary condition for such a protocol is the symmetry in the configuration: Two identical chaotic lasers should be coupled by identical mutual interactions.

In a face to face laser configuration (mutual coupling) where the setup is built symmetrically, isochronal synchronization, however, was always found to be unstable and one laser had to be slightly detuned to guarantee a well-defined leader and/or laggard configuration in order to achieve high fidelity synchronization [11–13].

In this paper, we present a configuration of two symmetric lasers that exhibit stable isochronal synchronization under symmetric operation conditions. The two lasers take equal roles in creating and maintaining synchronization without any symmetry breaking. Although the message transfer using this system has yet to be tested, this result is an important ingredient in the transmission of secret messages over a single public channel in both directions.

A schematic of our experimental system is shown in Fig. 1. Each laser receives a delayed signal from the other as well as a delayed self-feedback. The time delay between the lasers is denoted τ_c, the delay of the self-feedback is denoted τ_d, and the coupling and self-feedback rates are denoted σ and κ, respectively. The results presented in this paper are for $\tau_d = \tau_c = 7$ ns but synchronization was observed for other time delays as well. We measure the degree of synchronization by the time-dependent cross correlation [16], which is denoted as ρ and defined as follows:

$$\rho(\Delta t) = \frac{\sum_i (I_A^i - \langle I_A^i \rangle)(I_B^{i+\Delta t} - \langle I_B^{i+\Delta t} \rangle)}{\sqrt{\sum_i (I_A^i - \langle I_A^i \rangle)^2 \sum_i (I_B^{i+\Delta t} - \langle I_B^{i+\Delta t} \rangle)^2}}. \quad (1)$$

I_A and I_B are the time-dependent intensities of lasers A and B, respectively. We found it convenient to perform the experiments in a synchronization regime where total laser intensity breakdowns take place, commonly referred to as the LFF (low frequency fluctuation) [1–3] regime. In this regime, synchronization as evidenced by the correlated intensity breakdown of both lasers is easily observed. During the intensity breakdown, however, the system can temporarily desynchronize, as was proven numerically in Ref. [17]. In order to avoid these irregularities in our data analysis we divide the sequences into segments and we exclude segments containing a LFF breakdown. The correlation coefficient is calculated between matching time segments and then averaged over all segments. The time scale of the intensity fluctuations was of the order of 1 ns, which is also the experimental time resolution. For this reason we also averaged the simulation results over 1 ns, so that I_A^i is an averaged intensity for a window of 1 ns at time i. We arbitrarily chose the size of each segment to be 10 ns, which is an order of magnitude smaller than the average time between breakdowns—around 150 ns, and thus each segment consists of ten points (each point of 1 ns).

Figure 2 shows the shifted correlation coefficient $\rho(\Delta t)$ [9], which is obtained by calculating the correlation coefficient between the outputs of the two lasers when one is con-

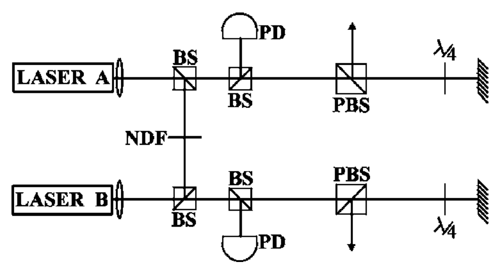

FIG. 1. A schematic figure of the two coupled lasers. BS, beam splitter; PBS, polarization beam splitter; NDF, neutral density filter; PD, photodetector.

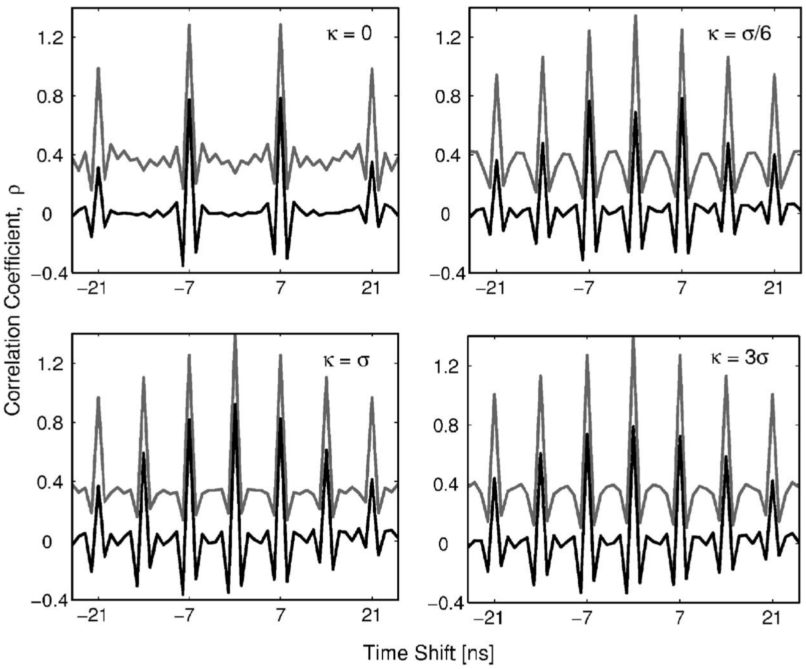

FIG. 2. Correlation coefficient between laser intensities at different time delays for $\tau_c = \tau_d = 7$ ns; κ, self-feedback rate; σ, coupling rate. The results of the experiment are plotted with a solid black line and the results of the simulations with a gray line, which has been shifted up by 0.4 for clarity. Different κ values affect the dynamics of the synchronization (in the simulations we kept $\kappa + \sigma = 100\ \mathrm{ns}^{-1}$). For $\kappa = 0$ (face to face configuration) we observe a coexistence of leader and/or laggard situation. When $\kappa > 0$ the isochronal synchronization is stable (maximum correlation in zero time delay).

tinuously shifted in time with respect to the other. The four graphs exhibit the dynamics of synchronization for different relations between coupling and self-feedback rates, σ and κ [19].

In all the graphs the symmetry is evident. Without self-feedback, when $\kappa = 0$, we find high correlation for time delays of $\pm\tau_c$ (7 ns) and no correlation at zero time delay, implying an achronal synchronization that was discussed in Ref. [12]. A further investigation of the symmetry of the achronal synchronization is given later in this paper.

One can clearly observe that in the other three cases displayed, for which $\kappa > 0$, there appears a very high correlation at zero time delay. For $\kappa = \sigma$, for instance, the experiment results show an average value of 0.92 and a most probable value of 0.99. In the simulations we report an even higher correlation which indicates complete synchronization in between the breakdowns. We can also see secondary peaks at $\Delta t = \pm n\tau_d$, where n is an integer. These peaks reflect the fact that the chaotic waveform has some self-correlation at time intervals of $n\tau_d$, where n is a small integer, due to the self-feedback. The above results hold, experimentally and in simulations, for the case of $\tau_c = \tau_d$. An interesting observation is that also for $\tau_c = n\tau_d$ where n is a small integer (up to about 3), stable isochronal synchronization appears. This is supposedly because of the self-correlation mentioned above.

In our numerical simulations we explored the phase space of κ and σ for $\tau_c = \tau_d$ as displayed in Fig. 3. Stable isochronal synchronization appears over a wide range of values of κ and σ (the dark gray circles in the graph). Without self-feedback the isochronal solution is unstable and the achronal synchronization appears indicated by the open circles. Achronal synchronization also appears when $\tau_c \neq \tau_d$.

We now give a more detailed description of the numerical simulations and the experimental results. To numerically simulate the system we used the Lang-Kobayashi equations [18] that are known to describe a chaotic diode laser. The dynamics of laser A are given by coupled differential equations for the optical field, E, the time-dependent optical phase, Φ, and the excited state population, n;

$$\frac{dE_A}{dt} = \frac{1}{2}G_N n_A E_A(t) + \frac{C_{sp}\gamma[N_{sol} + n_A(t)]^{-1}}{2E_A(t)} + \kappa_A E_A(t-\tau_d)\cos[\omega_0\tau + \Phi_A(t) - \Phi_A(t-\tau_d)] + \sigma_A E_B(t-\tau_c)\cos[\omega_0\tau_c + \Phi_A(t) - \Phi_B(t-\tau_c)],$$

$$\frac{d\Phi_A}{dt} = \frac{1}{2}\alpha G_N n_A - \kappa_A \frac{E_A(t-\tau_c)}{E_A(t)}\sin[\omega_0\tau + \Phi_A(t) - \Phi_A(t-\tau_d)] - \sigma_A \frac{E_B(t-\tau_c)}{E_A(t-\tau_c)}\sin[\omega_0\tau_c + \Phi_A(t) - \Phi_B(t-\tau_c)],$$

$$\frac{dn_A}{dt} = (p-1)J_{th} - \gamma n_A(t) - [\Gamma + G_N n_A(t)]E_A^2(t),$$

and likewise for laser B. The values and the meaning of the parameters are those used in Ref. [17].

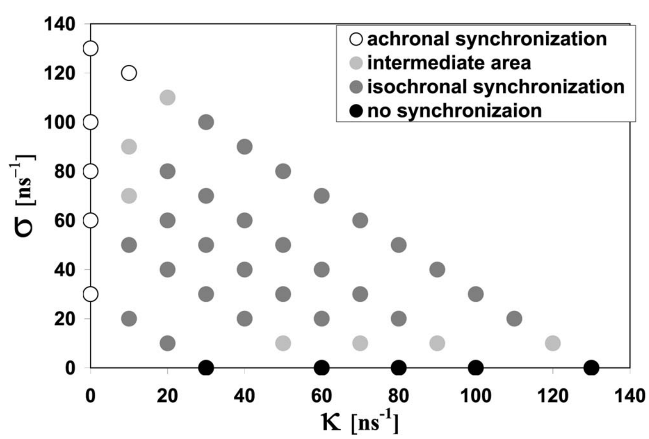

FIG. 3. The phase space of κ and σ, with $\kappa_A = \kappa_B$, $\sigma_A = \sigma_B$, and $\tau_c = \tau_d$ [20].

In the experimental setup we use two device-identical single-mode semiconductor lasers emitting at 660 nm and operated close to their threshold currents. The temperature of each laser is stabilized to better than 0.01 K. Both lasers are subjected to a similar optical feedback. The length of the external cavities is equal for both lasers and is set to 105 cm (round trip time τ_d=7 ns). The feedback strength of each laser is adjusted using a $\lambda/4$ wave plate and a polarizing beam splitter and is set to approximately 2% of the laser's power. It leads to a reduction of about 5% in the solitary laser's threshold current. The two lasers are mutually coupled by injecting another 2% of each one's output power into the other one. Coupling power is controlled by a neutral density filter. The coupling optical path is set to 210 cm (τ_c=7 ns) which is equal to the round trip length in the external cavities ($\tau_c=\tau_d$). Two fast photodetectors (with response time $<$500 ps) are used to monitor laser intensities, which are simultaneously recorded with a digital oscilloscope (500 MHz, 1 GS/s). Fine frequency tuning of the two lasers is crucial for the establishment of synchronization. This can be achieved by scanning one laser's temperature. While doing so we monitor both laser intensity signals on the digital scope. The desired temperature is attained when there is no obvious leader or laggard, i.e., both breakdowns occur simultaneously or within a time less than τ_c of each other and neither laser can be declared as the leader. The symmetry breaks a 0.02 K deviation from this temperature and one of the lasers turns into a laggard or leader, i.e., its signal seems to follow or precede the other laser's signal by a time τ_c. Whether the laser becomes a leader or a laggard depends very sensitively on the relative output powers of the two lasers.

Isochronal synchronization is established when the self-feedback round trip times (τ_d) of both lasers and the coupling optical travel time (τ_c) between them are all equal. In this type of synchronization we get a maximum overlap between the two signals for zero time delay between them. For fine tuning of the optical path lengths, two of the mirrors in the experimental setup were mounted on translation stages to simultaneously adjust τ_{dA} and τ_c to be equal to each other and to τ_{dB} which remained constant. We found that once high fidelity synchronization was established, even a change of 100 μm in the location of the mirrors caused the synchronization to deteriorate. This agrees with the results of the simulation that show that the synchronization is sensitive to small changes in τ_c and τ_d (see Fig. 4).

One of the challenging applications of chaotic lasers is in cryptographic systems. Unidirectional coupling was used for the creation of a secret-key cryptographic method [21], however, for the purpose of public-key systems such as a key-exchange protocol, one must use a symmetric system in which the two lasers have symmetric dynamics [22]. We therefore wish to discuss the symmetry of the synchronized states. The symmetry of the isochronal synchronization is nearly perfect, but the symmetry of the achronal synchronization, with $\kappa=0$, needs further investigation, as the correlation displayed in Fig. 2 is averaged. In our numerical simulations we observe that between the LFF breakdowns there is a high cross correlation both with delay $+\tau_c$ and $-\tau_c$, even without averaging over segments, as if the leading role is shared by the two lasers. Therefore, in the areas between the breakdowns there is considerable symmetry in the laser intensity sequences, whereas at the breakdowns, this symmetry is broken and the lasers exchange the leading role randomly between them, i.e., sometimes *A* falls before *B* and sometimes vice versa. Figure 5 displays a histogram of the ratio of the cross correlation with delay $+\tau_c$ and with delay $-\tau_c$, denoted as $\rho(+\tau_c)/\rho(-\tau_c)$. The peak at 1 indicates a high symmetry throughout the sequence. The inset of Fig. 5 displays a histogram of the time delay between the breakdowns of lasers *A* and *B*. The two peaks at $\pm\tau_c$ indicate that about half of the falls are lead by laser *A* and the other half by laser *B*. We conclude from these results that the achronal synchronization might be suitable for cryptographic purposes when excluding the LFF breakdowns, or in parameter regimes in which LFF breakdowns do not appear but the signal is still chaotic.

It is evident from the discussion above that for two mutually coupled lasers without self-feedback ($\kappa=0$), isochro-

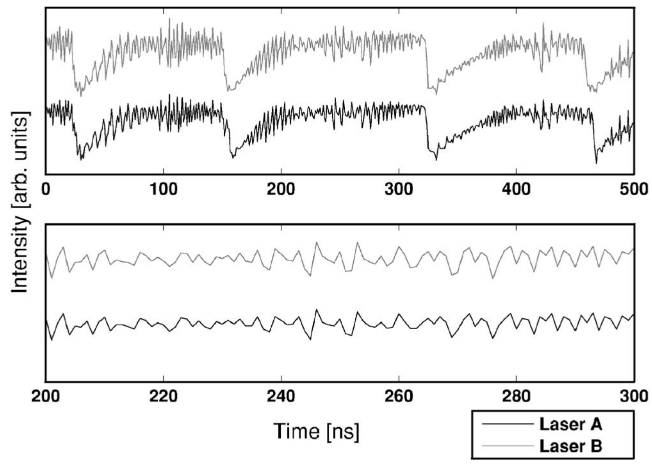

FIG. 4. A typical experimental time sequence of laser intensities for isochronal synchronization. The system parameters are $\kappa=\sigma$ and $\tau_c=\tau_d$. The lower figure is a more detailed view of the figure above it.

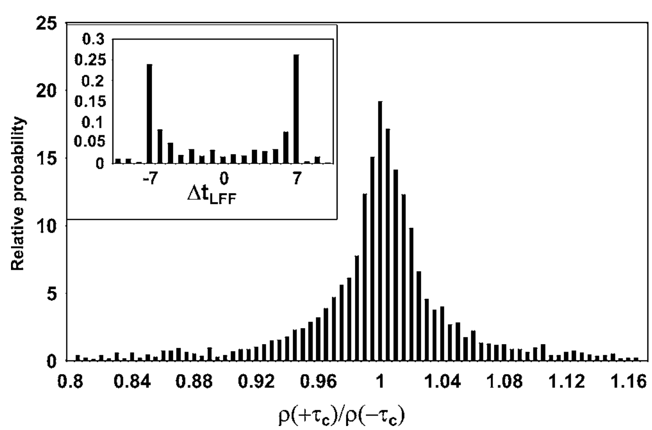

FIG. 5. Numerical simulations results for the achronal synchronization with $\kappa=0$, $\sigma=100\ \text{ns}^{-1}$, and τ_c=7 ns. The relative probability of the ratio of the cross correlation of two time delays $\rho(+\tau_c)$ and $\rho(-\tau_c)$. Inset: A histogram of the time delay between the breakdowns of the two lasers *A* and *B*, for the same parameters as above.

nal synchronization is not stable, yet by merely adding self-feedback, the isochronal synchronization becomes stable. This appears for a wide window of parameters. We do not prove here the stability or measure the distribution of Lyapunov exponents, but rather give an intuitive explanation for the difference between a system with and without self-feedback.

In a system without self-feedback each laser receives the delayed signal of the other laser. Starting from different initial conditions, even if after some time the lasers come very close to each other, they still have a different history which prevents them from completely synchronizing, because each laser continues to receive a different signal, $E_A(t-\tau_c) \neq E_B(t-\tau_c)$. Only if the two lasers reach a state in which their optical field and phase over an entire window of size τ_c is close enough, will they manage to remain synchronized. Otherwise, their different histories will drive them to achronal synchronization.

In a system with $\kappa > 0$, on the other hand, each laser receives the delayed signals of *both* lasers. Even if their history is different, the two lasers receive the same signal. It is then easier for them to remain synchronized because the difference in their delayed values does not affect the synchronization.

Another way to put this argument is by looking at one of the necessary conditions for isochronal synchronization. In order for isochronal synchronization to exist, the condition that $dE_A(t)/dt = dE_B(t)/dt$ must be satisfied, because the lasers' dynamics are first order differential equations. When $\kappa = 0$ and the lasers start in different initial states, this condition is only satisfied if $E_A(t) = E_B(t)$ holds for every time t over the interval $[t, t+\tau_c]$. However, when $\kappa > 0$ and $\kappa_A = \sigma_A = \kappa_B = \sigma_B$, then $dE_A(t)/dt = dE_B(t)/dt$ immediately follows. Therefore, a necessary condition for synchronization is more easily fulfilled for $\kappa > 0$.

In conclusion, the existence of a stable symmetric isochronal synchronization has been demonstrated in coupled lasers with self-feedback. In coupled lasers without self-feedback we have shown that partial symmetry appears in the form of achronal synchronization. The synchronization methods open the possibility of implementing optically chaotic systems in different communication tasks.

[1] J. Mork, J. Mark, and B. Tromborg, Phys. Rev. Lett. **65**, 1999 (1990).

[2] T. Heil, I. Fischer, and W. Elsaber, Phys. Rev. A **58**, R2672 (1998).

[3] Y. Takiguchi, Y. Liu, and J. Ohtsubo, Opt. Lett. **23**, 1369 (1998).

[4] C. R. Mirasso, R. Vicente, P. Colet, J. Mulet, and T. Perez, C. R. Phys. **5**, 613 (2004).

[5] J. Liu, H. Chen, and S. Tang, IEEE J. Quantum Electron. **38**, 1184 (2002).

[6] S. Donati and C. R. Mirasso, IEEE J. Quantum Electron. **39**, 1138 (2002).

[7] L. Larger and J. P. Goedgebuer, C. R. Phys. **5**, 609 (2004).

[8] S. Tang and J. M. Liu, IEEE J. Quantum Electron. **39**, 963 (2003).

[9] S. Tang and J. M. Liu, Phys. Rev. Lett. **90**, 194101 (2003).

[10] S. Tang, R. Vicente, M. C. Chiang, C. R. Mirasso, and J. Liu, IEEE J. Sel. Top. Quantum Electron. **10**, 936 (2004).

[11] T. Heil, I. Fischer, W. Elsasser, J. Mulet, and C. R. Mirasso, Phys. Rev. Lett. **86**, 795 (2001).

[12] J. Mulet, C. Mirasso, T. Heil, and I. Fischer, J. Opt. B: Quantum Semiclassical Opt. **6**, 97 (2004).

[13] J. K. White, M. Matus, and J. V. Moloney, Phys. Rev. E **65**, 036229 (2002).

[14] S. Sivaprakasam, E. M. Shahverdiev, P. S. Spencer, and K. A. Shore, Phys. Rev. Lett. **87**, 154101 (2001); M. C. Chiang, H. Chen, and J. Liu, IEEE J. Quantum Electron. **41**, 1333 (2005).

[15] E. Klein, R. Mislovaty, I. Kanter, and W. Kinzel, Phys. Rev. E **72**, 016214 (2005).

[16] H. Chen and J. Liu, IEEE J. Sel. Top. Quantum Electron. **10**, 918 (2004).

[17] V. Ahlers, U. Parlitz, and W. Lauterborn, Phys. Rev. E **58**, 7208 (1998).

[18] R. Lang and K. Kobayashi, IEEE J. Quantum Electron. **QE-16**, 347 (1980).

[19] The absolute values of σ and κ were difficult to attain experimentally, but we were able to give a good estimation as to the rate of self-feedback (κ) relative to the coupling rate (σ) by comparing the effect they have on the solitary laser. When subjected to each of them alone, it shows an increase in its output power.

[20] Note that the intermediate areas in which the isochronal synchronization is less stable, disappear in the case of $\kappa_A = \sigma_B$ and $\kappa_B = \sigma_A$ and instead there is stable isochronal synchronization for every $\kappa > 0$.

[21] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990); J. P. Geodgebuer, L. Larger, and H. Porte, *ibid.* **80**, 2249 (1998); G. D. VanWiggeren and R. Roy, Science **279**, 1198 (1998).

[22] N. Gross, E. Klein, M. Rosenbluh, W. Kinzel, L. Khaykovich, and I. Kanter, e-print cond-mat/0507554.